Currículo para o Século XXI

2017

Conferência Internacional

Pensar a Matemática

Teaching and learning elementary mathematics

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The current dilemma: three false dichotomies:

Managerialist: Global "competencies" *versus* "content"

(Mathematics is **the most important** "21st century competency")

Democratic: "Progressive left" *versus* "reactionary right"

Mathematics education: Child-centred *versus* subject-centred
"bottom-up" *versus* "top-down"

Experience: **"Think. Naively taking sides leads to trouble!"**

Learning and teaching mathematics more effectively is **hard**.
But – **given thought and cooperation**, it is possible.

Portuguese mathematicians and educators have to choose:
- To **work** (**together**) to develop the (mathematics) curriculum?
- Or to trust some external "21st century" *Snake Oil* – such as "definitions of concepts adopted by OECD"?

<u>TIMSS and PISA</u>: Learn. But **don't worship PISA scores.**
And don't just accept PISA's claims/OECD "competencies".

Portugal has seen dramatic change/improvement since 1995. The recent TIMSS/PISA results confirm this - though parties should hesitate to claim direct "responsibility" (England 2003-15)

Do not be seduced by **slogans**, or by reforms elsewhere (e.g. BC, Canada). The modern world is very different from the one I grew up in; but *bringing up children*, *learning language* and *reading*, internalising *place value*, *calculating with numbers*, with symbols, and with geometric shapes, etc. require one to go through *more-or-less the same stages as before*. In short:

- To access "21st century skills" one has to **start** with familiar 19th/20th century pathways.
  There is no "21st century short cut".

<u>Brief educational CV</u> 1968: New math for teachers in Tanzania
1975-82: Saturday maths school to reshape the curriculum.
1978-80: Visits to Freudenthal & IOWO (now often misrepresented).
1979-2000: *School Mathematics Project* – the only academic in a writing team for secondary textbooks (used in 60% of schools).
1980s: Start local and national secondary school "challenges".

My original goal: "Learn from other countries to find effective *alternatives to tradional instruction*."

What became increasingly clear:
- "Top-down" and "bottom-up" viewpoints must cooperate.
- Naive differentiation (acceleration for some; well-meaning bottom-up schemes for others) undermines progression <u>for all</u>
- Everyone can and must grasp the basic abstractions.

<u>An example very like Portugal</u>(P): namely UK/England(E).

- TIMSS Grade 4: P +100; E +50 points *since 1995*
- TIMSS Grade 8: P improves slightly; E improves **not at all**

England Huge effort at primary level (2000) – and real "change". But *purely "bottom-up", so "backward-looking".* We failed to identify and **lay the foundations for secondary mathematics**. (Wales chose a different, gentler route: "interdisciplinary", "projects", etc. Result: **very, very bad news for Wales!**)

A balanced curriculum has to focus on achieving serious mathematical goals at age 16/18 **for large numbers**. So it has to respect **the structure of mathematics**, *while also respecting the way young minds internalise abstract mathematics.*

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At primary level in England, the most striking example of this combined "top-down + bottom-up" approach is to be found in

- the links with Shanghai , and
- the primary *Singapore textbooks project*, which is having a dramatic impact on both pupils **and teachers.**

Both are helping us to rediscover *didactics* (how to prepare the ground; how to introduce ideas/methods; how to use a few consistent models to support cumulative internalisation).

The approaches

- are "open" (but focused to establish the "target method")
- are "universal" (whole classes, very little differentiation)
- emphasise simple structures, in the maths and the learning
- focus on "variations" (what is needed for progression).

Neither approach has been extended beyond Grade 5.

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Key ingredients of any successful approach (?)

- Classes are best taught as a (social/communal) whole.

- Key ideas are accessible to **all** if taught in a structured way.

- Mathematics is inescapably **abstract**, *but does not start thus*: [hence the C-P-**A** approach, with C-P as stepping stone to **A**]

- One cannot separate "process" from content: **there are no "competencies", or "21st century skills" that can be taught separately from mathematics.** Hence the first task is

  • to organise *the mathematical/didactical sequence*
  • then experiment to find effective ways to teach it.

- Find a didactical structure that works, and develop it (most additions, such as "pedagogical differentiation, experimental activities, projects, and interdisciplinary work" make it **hard** for pupils/teachers to see **where they are going**.

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Content and style

There is widespread agreement about much **early** content;
yet it is surprisingly hard to draft a *coherent curriculum*.
This is partly due to the superficial understanding of many teachers, educators and mathematicians as to what aspects of each topic are **essential for subsequent progression**.

This superficiality becomes **more marked at secondary level**.

Nevertheless, one can draft a potential curriculum as a basis for discussion (as indicated by Part III of my book:
Teaching mathematics at secondary level, Open Book, 2014).

"Top-down" mathematicians may try to hurry certain stages; and "bottom-up" educators may need to reconsider the importance of themes they are inclined to dismiss.
But they must work together to reach a consensus.

What **cannot** work is

(i) to add more content than can be effectively introduced, understood, and mastered in the available time, or

(ii) to take a carefully planned (but perhaps too demanding) sequence and simply "cut 25%" to make more time available.

The mathematics curriculum is not an expanding suitcase into which more and more material can be stuffed.
Nor is it a piece of string, which can be shortened by cutting.

It **is a structure** that can be extended or slimmed down - but must then be redesigned. Hence one can try to reduce the curriculum – but be prepared to discover that some of what you try to remove is *essential for parts that you wish to retain*.
Some strands may have been included early for bad reasons (e.g. Data), may now be seen to be premature, so can be revised/delayed.

Language: Early years mathematics, and its associated logic, have their roots in language: the "**C**" in **C**-A-P often involves focused group **discussion** of an example or problem situation. And at every stage, the link between mathematics and language needs to be strengthened through *Word Problems*.

Number and measures: All pupils need to master the "place value" system. This requires that they become friends

- with the numbers 1-5, then 1-10 (and "0")
- with the associated number-words, and
- with *combinations/differences; addition/subtraction*

later using these as digits in numeral form (29; 4562; 7,093).

The next sequence

- multiplication/division (including written algorithms), fractions, decimals, negative numbers, powers/roots/surds, irrationals, ...

needs to be carefully sequenced – linking with **measures** from the outset, and preparing for **algebra**.

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Structural arithmetic: Calculation needs to exploit place value and to become robust and varied. Upper primary must move beyond "blind calculation with larger and larger numbers" to focus on **structure**

– exploiting place value ($73 + 39 + 27 = ?$, $12 \times 75 = ?$, ...) and

– algebraic structure ($7\times29 + 3\times29$, $3^2 \times 4^2$, $0.144 \div 1.2$, ...)

in preparation for **algebra**.

Elementary number theory Work with fractions and structure draws attention to squares, cubes; factors, common factors; primes, prime factorisation; etc. (to be handled in the spirit of a sensitive teacher rather than a pure mathematician!)

Measures and geometry: Measures are central to number, but also feature in geometry, which has a dual nature – part numerical, and part purely geometrical. How this is treated requires a consensus about the secondary mathematics to which it leads (fractions, algebra, functions and graphs, euclidean geometry, etc.).