Energizers and hindrances for the design and implementation of mathematics curricula

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Hindrances

A metaphor for my talk

Energizers
A metaphor for my talk

Hindrances

Ambiguities

Energizers
Overview

XXI century society: a challenge for curricula

A multidimensional framework for curricula

Energizers for acting on teachers’ beliefs:
- Technology
- Assessment
- Teachers’ education programs

Hindrances for learning:
- Technology
- Gender gap in Mathematics

Paideia 2.0: an example
Liquid Times:
• Constantly changing conditions
• Uncertain future
• Collapse of long-term thinking
• Focus on short-term goals
• Focus on individual responsibility
• Risk is to stand still

XXI century society: a complex challenge for curricula
- **Pointillist time**
- **Nowist culture**: dissolution of the plot that connects the present with the past and with the future
- **Lack of narratives**: the fragments threaten to become hegemonic.
Too much information = Zero information
Towards a παιδεία 2.0
(= paideia + “liquid” practices)

- Against pointillist time
- Reconstructing narratives
- Reconstructing intersubjective links
- Using the ICT affordances for surfing through the liquid society
Liquid practices

ICT (e.g. Mobiles) affordances to enable learning in liquid society:

- Accessing
- Sharing
- Building
- Co-creating
- Supporting
- Managing
- Across settings
- Across time

(Franziska Trede, 2016)
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Paideia 2.0: an example
A multidimensional Framework

Levels

MACRO

MICRO

Facets

Intend. Textb. Implem Assess. Learned ...

A multidimensional Framework
A multidimensional Framework

Levels

MACRO

MICRO

Topics

Facets

Intend.  Textb.  Implem.  Assess.  Learned  ...

REPÚBLICA PORTUGUESA
EDUCAÇÃO
The dimensions of the curriculum

How to use them?

In designing
In comparing
In focussing

Curricola

Levels

MACRO

MICRO

Topics

Facets

Intend, Textb, Implement, Assess, Learned
1a. Designing Curricula: a complex task
What /How to teach?
The development of a school curriculum must take into account both the instrumental and the cultural function of mathematics: it is an essential instrument for a quantitative understanding of reality and logically coherent and systematic knowledge characterised by a strong cultural unity.
The teacher is supposed to tackle these themes in an integrated manner, trying to connect them to other topics and to other subject disciplines.

Many examples are necessary to give the sense of the integration of the three structural components.
1b. Comparing Curricula: the risk of ambiguity
The structural description of curricula can be an antidote against possible mistakes done when analysing curricula for different purposes: comparisons, assessments, content, ...

The complexity of the structure shows how complex any analysis should be and how easy can be drawing superficial or wrong conclusions because some level or facet is forgotten.
As E. Silver (2009) points out, the intentions and actions of the consumers of international comparisons often rest on too simplistic assumptions about the relationship among the various interactive constituent parts of the education system in a country: official curriculum goals, textbooks used in schools, teaching practices, teacher preparation and ongoing support, and student learning outcomes.
This superficial approach can have negative consequences: for example policy makers could be induced to undertake programs and give recommendations, which rest on partial or misinterpreted data.
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The compartmentalization of curriculum components

How to connect?
Energizers to connect the different components

Technology
Assessment
Teachers’ Education Programs

Teachers’ Beliefs
Without the contribution of teachers to the elaboration of curricula there is no change in the implemented curriculum.
A specific frame: Teachers’ Beliefs and Curricula
It is well known that there can be a great difference between the adopted curriculum and the intentions of people or institutions, who designed it, and the ways according to which the curriculum materials (e.g. textbooks) are concretely enacted in the classrooms. The influence of teachers’ beliefs on this gap between the various facets of curricula is remarkable (e.g. see Kulm & Li, 2009).
Many studies, basing on PISA and TIMSS data, show that, notwithstanding the emphasis given in many curricula to high-level cognitive processes (e.g. reasoning and problem solving), the believes of teachers about the effective ways they can teach mathematics to “mean achievers” students are at the origin of the limited opportunities they give to their students in such processes in their lessons (Silver, 2009).
These beliefs persist in the schools notwithstanding many researches show that a better learning occurs exactly in those classrooms where teaching is based on high-level cognitive demands and not only on stressing procedural instruction.
Research conducted in the past decade or more in a variety of different classroom contexts has found that greater student learning occurs in classrooms where the high-level cognitive demands of mathematical tasks are consistently maintained throughout the instructional episode.


Energizers to connect the different components

2a. Technology
2b. Assessment
2c. Teachers’ Education Programs
The Technology Principle of NCTM Principles & Standards

Technology:
- enhances mathematics learning;
- supports effective mathematics teaching;
- influences what mathematics is taught.
Towards the digital school!
Energizers to connect the different components

2a. Technology
2b. Assessment
2c. Teachers’ Education Programs
CONS: The risk of an education test-oriented

PROS: Possible positive effects in schools’ practices
PISA Mathematics mean scores for the five areas of Italy
Energizers to connect the different components

2a. Technology
2b. Assessment
2c. Teachers’ Education Programs
1. 2001-2005: *Mathematics for the Citizen*: an intended curriculum with 200 examples of teaching situations (→ textbook curriculum)

2. 2006-2015: *M@t.abel*: interactive online activities with teachers for improving the implemented curriculum


4. 2015: Towards the digital school
La Matematica per il cittadino

Attività didattiche e prove di verifica per un nuovo curricolo di matematica

Quinta classe del ciclo secondario di secondo grado
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Paideia 2.0: an example
Figure 4.5a: Frequency of use of computers at home and student performance in mathematics

- Rare or no use
- Moderate use
- Frequent use

PISA score points

Proficiency levels
- Level 4
- Level 3
- Level 2
- Level 1

Countries: Finland, Japan, Liechtenstein, Korea, Slovak Republic, Czech Republic, Canada, Latvia, Iceland, Ireland, Russia, Denmark, New Zealand, Sweden, Switzerland, Australia, Poland, Austria, Germany, Belgium, Hungary, Serbia, United States, Portugal, Italy, Greece, Turkey, Uruguay, Thailand, Mexico, Tunisia, United Kingdom.
...however
PISA shows that even when most students have easy access to new media, inequalities persist in the way they use these tools. The use of online media depends on the student’s own level of skills, motivation, and support from family, friends and teachers, which vary across socio-economic groups.
Ensuring that every child attains a baseline level of proficiency in reading will do more to create equal opportunities in a digital world than will expanding or subsidising access to high-tech devices and services.
A hindrance variable in mathematics curriculum development: the gender gap
Boys keep doing better than girls in math tests. According to PISA, the average gender differential within OECD countries in mathematics at age 15 is 0.11 standard deviations in favour of males (OECD 2015).
### Exhibit 1.10: Average Mathematics Achievement by Gender

<table>
<thead>
<tr>
<th>Country</th>
<th>Girls</th>
<th></th>
<th>Boys</th>
<th></th>
<th>Difference (Absolute Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent of Students</td>
<td>Average Scale Score</td>
<td>Percent of Students</td>
<td>Average Scale Score</td>
<td></td>
</tr>
<tr>
<td>Hong Kong SAR</td>
<td>46 (1.5)</td>
<td>609 (3.8)</td>
<td>54 (1.5)</td>
<td>619 (2.8)</td>
<td>10 (3.3)</td>
</tr>
<tr>
<td>Portugal</td>
<td>49 (0.8)</td>
<td>536 (2.4)</td>
<td>51 (0.8)</td>
<td>547 (2.5)</td>
<td>11 (22)</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>48 (0.9)</td>
<td>493 (3.0)</td>
<td>52 (0.9)</td>
<td>504 (2.6)</td>
<td>11 (26)</td>
</tr>
<tr>
<td>Spain</td>
<td>49 (0.9)</td>
<td>499 (2.7)</td>
<td>51 (0.9)</td>
<td>511 (2.7)</td>
<td>12 (24)</td>
</tr>
<tr>
<td>Croatia</td>
<td>49 (0.8)</td>
<td>496 (2.1)</td>
<td>51 (0.8)</td>
<td>508 (2.3)</td>
<td>12 (27)</td>
</tr>
<tr>
<td>Italy</td>
<td>49 (0.7)</td>
<td>497 (2.7)</td>
<td>51 (0.7)</td>
<td>517 (3.0)</td>
<td>20 (27)</td>
</tr>
<tr>
<td>International Avg.</td>
<td>49 (0.2)</td>
<td>505 (0.5)</td>
<td>51 (0.2)</td>
<td>505 (0.5)</td>
<td></td>
</tr>
</tbody>
</table>
Figure I.5.10 • Gender differences in mathematics performance

Score-point difference in mathematics (boys minus girls)

Mean score in mathematics

<table>
<thead>
<tr>
<th>Country</th>
<th>Score</th>
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<tbody>
<tr>
<td>OECD average</td>
<td>490</td>
</tr>
<tr>
<td>Hungary</td>
<td>477</td>
</tr>
<tr>
<td>Israel</td>
<td>470</td>
</tr>
<tr>
<td>United States</td>
<td>470</td>
</tr>
<tr>
<td>New Zealand</td>
<td>495</td>
</tr>
<tr>
<td>Canada</td>
<td>516</td>
</tr>
<tr>
<td>Denmark</td>
<td>511</td>
</tr>
<tr>
<td>Kosovo</td>
<td>362</td>
</tr>
<tr>
<td>Peru</td>
<td>387</td>
</tr>
<tr>
<td>Portugal</td>
<td>492</td>
</tr>
<tr>
<td>Colombia</td>
<td>390</td>
</tr>
</tbody>
</table>
The presence of a substantial females’ disadvantage in math is of particular importance, because it is likely to be a cause of the critically low share of women choosing STEM disciplines at university, of gender segregation in the labour market, and gender pay gaps (European Commission 2006, 2012, 2015).
The main finding from researches in USA about primary school (confirmed by PISA surveys at 15 years) is that the math gender gap starts as early as in kindergarten and increases with the age of the child.
Another relevant result is that the math gender gap is higher for top performing students. Initially boys appear to do better than girls among well performers and worse at the bottom of the distribution. By third grade, the gender gap, still larger at the top, appears throughout the distribution.
Survey responses regarding self-concept in maths (year 5 and 6) and on the importance of math for their future life (year 10) show that boys are substantially more confident on their own abilities than girls are, and that they are more aware of the importance in math for their future.
Reducing the gender gap in STEM education and increasing the number of women graduating in STEM subjects leads to an increase in labour supply, and in employment. This could help reduce bottlenecks in the labour market.
Effect of closing the educational gap on GDP per capita
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Paideia 2.0: an example
What kind of mathematics in παιδεία 2.0?

An example
Modelling with mathematics
Change is crucial from many standpoints:

- **Cognitively**: it draws attention;
- **Epistemologically**: its analysis is the root of the scientific revolution → **Calculus**
- **Culturally**: understanding it in climate, economy,,… is a crucial issue in XXI cent. society;
- **Didactically**: Finite differences are a powerful tool, which can easily be implemented with didactical software and allow modelling a variety of phenomena from early grades.
A finer idea of change

$$\Delta A = 5 \text{ cm}^2$$

The relative change $$\Delta_r A = \Delta A / A$$

$$\Delta_r = \frac{5 \text{ cm}^2}{5 \text{ cm}^2} = 100\%$$

$$\Delta_r = \frac{5 \text{ cm}^2}{50 \text{ cm}^2} = 10\%$$
Relative differences: polynomes

$$y = a \ n^b$$

$$f(n) = 2 \ n^{1.6}$$

$$g(n) = \Delta f$$

$$h(n) = \Delta f/f$$

<p>| | | | | | | |</p>
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<td>D</td>
<td>E</td>
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<tr>
<td>1</td>
<td>n</td>
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<td>n</td>
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<td>22.73</td>
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</table>

INSTR.
Relative differences: exponentials

\[ y = c \ a^{(b \ n)} \]

\[ f(n) = 0.4 \ 2^{(0.4 \ n)} \]

\[ h(n) = \Delta f/f \]
Growth phenomena in Biology and Economy: reasoning about changes as education to rational decisions

\[ f(x) = \frac{a}{d + \exp(-b(x-c))} \]

\[ g(x) = f'(x) \]

\[ h(x) = \frac{f'(x)}{f(x)} \]

http://mediarepository.indire.it/iko/uploads/allegati/M7PWITOE.pdf
The moral of my talk:

To break hindrances and transform them into energizers we need a critical attention to the ongoing changes in society and a careful action on teachers and students beliefs. Otherwise also the best intended curriculum will produce little change in the implemented and learned ones.
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